2007 – 2008 Log1 Contest Round 3
Individual Answers

Theta Answers	
1	1587
2	4 [hours]
3	$4\pi - 8$
4	$\frac{7}{24}$
5	$\frac{53}{30}$
6	30
7	$FFF_{16}=4095$
8	$= \pi^2 + 5\pi$
9	2
10	0.5 [minutes] or 30 seconds
11	78%
12	0
13	$\pi\frac{\sqrt{35}}{3}$
14	8
15	60

Alpha Answers	
1	1
2	4 [hours]
3	$4\pi - 8$
4	$\frac{4\pi}{9}$
5	$x = -4 \text{ and } -\frac{1}{3}$
6	30
7	$FFF_{16}=4095$
8	$-\frac{119}{169}$
9	302400
10	0.5 [minutes] or 30 seconds
11	5
12	$\frac{1}{3}$
13	$\pi\frac{\sqrt{35}}{3}$
14	8
15	60

Mu Answers	
1	1
2	24x+14
3	$4\pi - 8$
4	$\frac{4\pi}{9}$
5	$x = -4 \text{ and } -\frac{1}{3}$
6	-108
7	$FFF_{16}=4095$
8	$-\frac{119}{169}$
9	302400
10	-1
11	5
12	$\frac{1}{3}$
13	$\pi\frac{\sqrt{35}}{3}$
14	90000
15	$\frac{22\pi}{3}$

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Individual Solutions

Mu	Al	Th	Solution
		1	The interior measure of a angle of a regular decagon using the exterior angle = (180 – 360/10)=144. 144x11+3=1587.
1	1		tan(0)=0 and sin(0)=0 and cos(0)=1.
	2	2	Eric works at a rate of 1/1 and Trevor at a rate of 1/2 so together they work at the rate of 3/2, so to do 6 tests it takes 6 / (3/2) = 4 hours.
3	3	3	The area of the circle is $\pi 2^2 = 4\pi$. The diameter 4 is the diagonal of the square. The area of the square is ½ the product of the two diagonals = ½ (4)(4)=8. Now subtract $4\pi - 8$
		4	
4	4		The circumference of the whole circle is $2\pi(4) = 8\pi$. The portion included in the arc is $\frac{20}{360} 8\pi = \frac{4\pi}{9}$
		5	A common denominator is 30. $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}=\frac{15}{30}+\frac{20}{30}+\frac{18}{30}=\frac{53}{30}$
5	5		One can either use the quadratic formula or use grouping to factor: (3x+1)(x+4) giving the roots -4 and -1/3
	6	6	$(x+y)^2 = x^2+2xy+y^2$, so $4^2 = x^2+y^2+2(3)$ and $x^2+y^2=10$. $3x^2+3y^2=30$
2			The first derivative is $12x^2+14x+11$ and the second derivative is $24x+14$
6			Add the first row to the second getting: $\begin{bmatrix} 6 & -8 & 5 \\ 0 & -6 & 1 \\ 9 & 3 & 8 \end{bmatrix}$ and expand along the first column: 6[(-6)(8)-(3)(1)] – 0[] + 9[(-8)(1)-(-6)(5)] = -108.
7	7	7	$FFF_{16} = 1000_{16} - 1 = 16^3 - 1 = 4095$
		8	$f(\pi+1) = (\pi+1)^2 + 3(\pi+1) - 4$ $= \pi^2 + 2\pi + 1 + 3\pi + 3 - 4$ $= \pi^2 + 5\pi$
8	8		$\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\frac{144}{169} = -\frac{119}{169}$
		9	This can be factored by grouping: $x^2(5x+1) - 1(5x+1) =$ $(x^2-1)(5x+1) = (x-1)(x+1)(5x+1)$ so the roots -1 and +1 have the largest difference.
9	9		There are 10 letters with 3 I's, and 2 D's so the number of distinct permutations is $\frac{10!}{3!2!} = 302400$
	10	10	The expected wait = (5 lights)*(2 min) * (.25 prob) = 2.5 minutes. Adding the 12 min. gives a .5 minute cushion.
10			The fifth roots of 1 are the roots of the polynomial $x^5 - 1 = 0$. The sum of all the roots is 0. Since 1 is the only real root, the complex ones must add to -1.

Mu	Al	Th	Solution
		11	$LW = (1.3l)(.6w) = .78lw$
11	11		One can substitute each of the points into the parabola and get three equations in A, B and C. Or one can realize that two of the points have the same y-coord. and since a parabola is symmetric the vertex must be at x=1 so the equation must be $y = a(x-1)^2 + b$. Plug two of points to get equations for a and b giving a=3 and b=5. Multiply this out to get A+B+C=5.
		12	One could try graphing but to be sure set the y-coordinates equal to each other and solve. $3x - 10 = x^2 + 14x + 50$ $x^2 + 11x + 60 = 0$ which has a negative discriminant and only complex roots.
12	12		Taking the terms three at a time, the new series is: $\frac{1}{4} + \frac{1}{32} + \frac{1}{256} + \ldots$ and the sum is $\frac{1/4}{1-1/8} = \frac{2}{7}$
13	13	13	60° is 1/6 of the circle so the arc which will become the circumference of the base is $\frac{1}{6}(2\pi)(6) = 2\pi$ so the radius of the base is 1. The slant height is 6, so by Pythagoras the height is $\sqrt{35}$ and the volume is $\pi\frac{\sqrt{35}}{3}$
	14	14	Let j=2n+1 and k = 2m+1. $j^2 - k^2 = 4(n^2+n) - 4(m^2+m)$. n^2 and n are either both even or both odd making their sum even; so $4(n^2+n)$ is divisible by 8. The difference between two multiples of 8 is also a multiple of 8.
14			Let x be the additional length along the wall and y the width. The perimeter of fencing is then y+x+y+(x+200)=1000 so x+y=400, y=400-x and the area is (400-x)(200+x) which is maximized when x = 100. The maximum area is then (100+200)(400-100) = 90000.
	15	15	Let the two equal sides be equal to x and the base equal to 2y. The perimeter 2x+2y=50 or x+y=25. By Pythagoras, $25=x^2-y^2=(x+y)(x-y)$. So x-y=1 and y = 12. The area is then 5x12=60.
15			Using the disc method and using horizontal discs, the volume is then $\int_{-1}^{1} \pi\left(\sqrt{4-y^2}\right)^2 dy = \pi\left(4y - \frac{y^3}{3}\right)\Big\|_{-1}^{1} = \frac{22}{3}\pi$